Experiments on Sound Radiation from a Duct with a Circumferentially Varying Liner

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The radiation of sound from an asymmetrically lined duct is experimentally studied for various hard-walled standing mode sources. Measurements were made of the directivity of the radiated field and amplitude reflection coefficients in the hard-walled source section. These measurements are compared with baseline hard-wall and uniformly lined duct data. The dependence of these characteristics on mode number and angular location of the source is investigated. A comparison between previous theoretical calculations and the experimentally measured results is made and, in general, good agreement is obtained. For the several cases presented, an asymmetry in the liner impedance distribution was found to produce related asymmetries in the radiated acoustic field.

Nomenclature

a = duct radius, m
c = speed of sound, m/s
i = $\sqrt{-1}$
k = free wavenumber, ω/c, m^{-1}
k_n^r = radial wavenumber, m^{-1}
k_{mn} = axial wavenumber, $k_{mn}^2 = k^2 - (k_n^r)^2$, m^{-1}
ℓ = length of lined section, m
m,n = mode number, (0,0) is plane wave
p = acoustic pressure, N/m^2
P_{mn} = pressure amplitude, N/m^2
r = radial cylindrical coordinate, m
R = amplitude reflection coefficient, nondimensional
t = time, s
x = axial cylindrical coordinate, m
θ = angular cylindrical coordinate, rad
ϕ = phase, deg
ψ = radiation angle, deg
ω = frequency, rad/s

Introduction

THE last twenty years have seen a dramatic increase in the volume of air traffic. Concern over the associated problem of environmental noise produced by these aircraft operations has spawned an extensive research effort to control and reduce its impact. In dealing with the forward radiated noise, the majority of research has been concerned with attenuation of sound as it propagates *within* the turbofan inlet.[1] Although in-duct attenuation is of prime importance in controlling the noise of turbofan engines, so too is any advantage that can be taken of the inlet radiation characteristics. Ultimate noise annoyance will depend upon the levels of sound radiated at angles toward the ground. Thus, in the last decade researchers have investigated the effects of inlet liner wall impedance,[2-5] flow of the propagating medium,[6] and duct inlet geometry[7] on the radiation characteristics of aircraft engine inlets. The results of these investigations have generally demonstrated the importance of including radiation effects in the analytical model.

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Previous theoretical work[8] has predicted that significant distortion can be created in the acoustic field radiated from an asymmetrically lined duct. The implication of this analysis is that by designing a liner to be nonuniform, it is theoretically possible to achieve a preferred directivity of the radiation field so as to reduce noise levels heard at ground receiving points. It is thus the aim of this investigation to experimentally examine the radiation behavior of an asymmetrically lined duct relative to that of hard-wall and uniformly lined ducts in order to establish any advantage of this configuration and also to provide a verification of the predictions of Ref. 8.

The experimental system utilizes NASA Langley's spinning mode synthesizer to generate circumferential standing waves of various modal orders to propagate through hard-wall, uniform, and asymmetric liner geometries and radiate to the far field from a flanged outlet. Measurements were made of the incident, reflected, and radiated acoustic fields. Comparions are drawn between the experimental results and the theory of Ref. 8.

Experimental Layout and Procedure

The layout of the experimental equipment is shown as a plan view in Fig.1. The principal piece of equipment is a scaled-down version of NASA Langley's spinning mode synthesizer (described in detail in Ref. 7), which is used to generate individually dominant circumferential standing waves. This apparatus, installed in NASA Langley's Anechoic Noise Facility, is capable of exciting circumferential mode orders up to $m=6$ at frequencies up to 5000 Hz in an 0.15 m radius duct. For this investigation, the modal orders considered were $m=0$-4 in the nondimensional frequency range of $ka=1$-10. In each case, the incident wave was a circumferential standing mode whose orientation was fixed with duct angle θ. Thus the incident wave in the hard-walled duct upstream of the lined section was of the form,

$$P^+(x,r,\theta,t) = P_{mn}^+ \begin{Bmatrix} \cos(m\theta) \\ \sin(m\theta) \end{Bmatrix} J_m(k_n^r r) \exp[i\omega t - ik_{mn}x] \quad (1)$$

where k_{mn} and k_n^r are axial and radial wavenumbers, respectively. The orientation of the source was chosen such that for even values of m the angular distribution was $\cos(m\theta)$ (relative to the line of impedance discontinuity) and for odd values of m the angular distribution was $\sin(m\theta)$. As discussed in Ref. 8, this particular selection of the angular location of the source ensured that the least attenuated mode was always excited most strongly in the asymmetricaly lined duct.

The plan view of Fig. 1 shows that the spinning mode synthesizer is coupled to a source measurement section where a careful mapping of the in-duct acoustic field allows a definition of the incident mode of propagation. The analysis technique to perform the forward-backward mode decomposition utilizes the Fourie-Bessel transform method outlined in Ref. 9. This method decomposes the acoustic field into positive and negative spinning modes. As this experiment is for circumferential standing modes of propagation, it is necessary to recombine the positive and negative spinning components of each mode to obtain standing wave amplitudes and phases. The reflection coefficient of the discontinuity caused by the lined section and the flanged outlet in series can be calculated from the measured complex amplitudes as

$$R = |P_{mn}^{-} / P_{mn}^{+}| \tag{2}$$

where P_{mn}^{-} and P_{mn}^{+} are the amplitudes of the reflected and incident waves for a (m,n) standing wave source. Likewise, the phase change of the incident wave at the liner inlet may be calculated by using the axial phase speed of the (m,n) mode to reference the phases measured at the decomposition plane to phases at the liner inlet plane.

The source measurement section then joins to the test section. The test section is so constructed that hard-walled, uniformily lined, and asymmetrically lined configurations may be installed. The end of the lined section terminates on a 2.4 m diameter baffle located in the wall of the 7.3 m cubic anechoic room in which the radiation directivity is measured on a horizontal arc of 2.7 m radius centered on the duct outlet. Figure 2 is a photograph of the flanged duct outlet and the far-field microphone traverse seen from inside the anechoic chamber. Although an unflanged duct would be a better model of a turbofan inlet, the flange is used to accurately represent the analysis of Ref. 8. The asymmetry of the liner is arranged so that the far-field microphone traverses in a plane from the middle of the soft-walled side ($\psi < 0$) to the middle of the hard-walled side ($\psi > 0$) in order to measure the greatest distortion of the radiated field. The end of the duct on the opposite side of the source to the flanged outlet is anechoically terminated in order to prevent any energy scattered into other circumferential modes from being re-reflected and contaminating the incident wave. For the examples discussed in this paper, the desired incident mode dominated all other modes in the hard-walled duct section by better that 15 dB.

The end-view geometry of the liner configurations and the impedance characteristics of the liner material used in this experiment are given in Figs. 3 and 4, respectively. For comparison purposes, the cases of a completely hard-walled section and a uniformly lined section were also investigated.

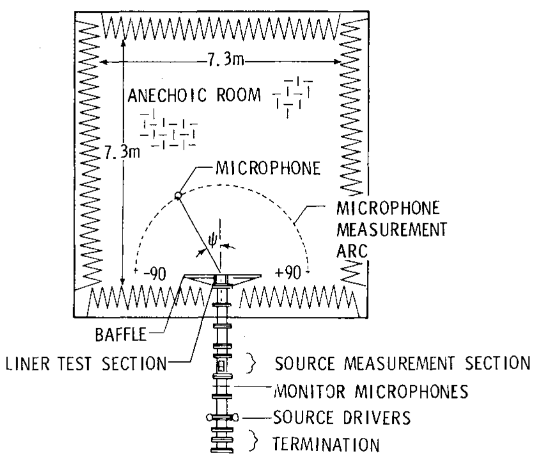

Fig. 1 Plan view of the experimental apparatus.

The liner material is a foam type bulk absorber arranged in 7.6 cm^2 compartments in order to reduce any extended reaction effects. The cavity depth of the liner was fixed at 4.6 cm and two lengths of liner corresponding to one and two radii ($\ell/a = 1$ and 2) of the duct section were investigated. A photograph of the asymmetric liner configuration is shown in Fig. 5 and illustrates the liner construction as well as the nature of the circumferential impedance discontinuity.

All the experimental hardware was computer controlled using a data acquisition system described in detail in Refs. 7 and 9.

Experiment Results and Discussions

Figures 6, 7, and 8 show experimental results in which distortion was observed in the acoustic field radiated from the asymmetrically lined duct. The results are presented so as to be normalized to the maximum pressure radiated from the completely hard-walled case. The amplitude of the incident wave was reduced from the in-duct data and scaled to a common level for each test.

Figure 6 presents typical measured radiation patterns. In this case, the incident wave is a (3,0) mode at $ka = 7.2$. The length of the lined section is $\ell/a = 2$. Marked distortion is observed in the radiated field for the asymmetrically lined case, due mainly to suppression of the principal radiation lobe in the positive ($\psi > 0$) sector of the radiated field. For this situation, a reduction in sound levels is observed in that part of the field diagonally opposite the lined region of the duct as would be expected from a simple ray theory approach. The asymmetric liner can be seen to provide the same order of attenuation in the positive sector of the radiated field as the uniform liner, and this is generally the case for all higher order circumferential modes well above cut-on.

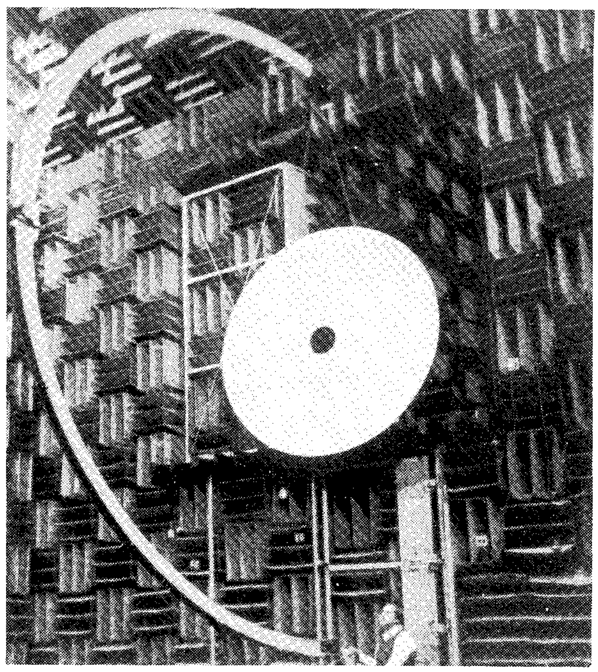

Fig. 2 The outlet baffle and far-field traverse.

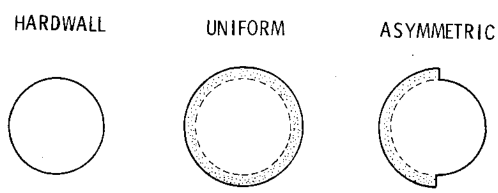

Fig. 3 Liner geometries.

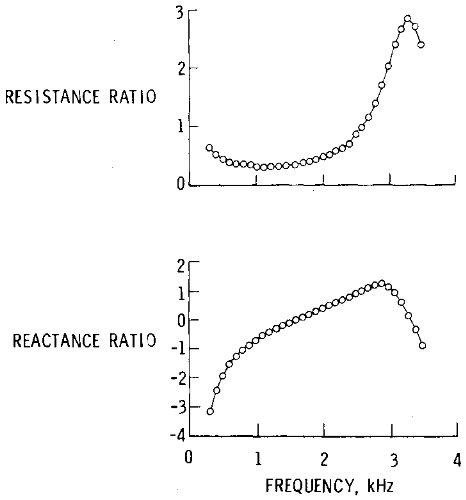

Fig. 4 Impedance characteristics of the liner material.

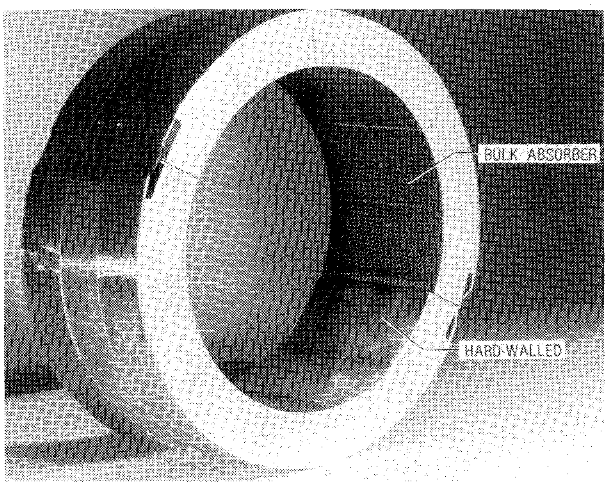

Fig. 5 Configuration of the asymmetric liner.

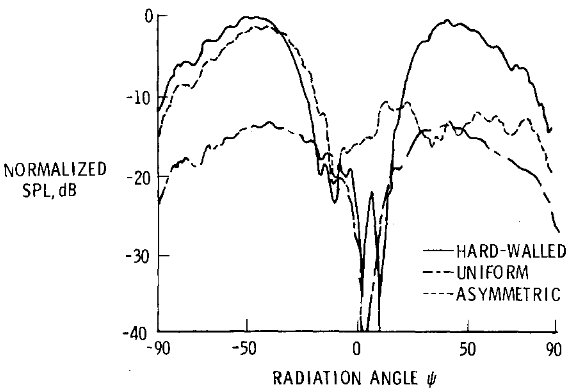

Fig. 6 Directivity patterns for a (3,0) mode, $ka=7.2$, $\ell/a=2$.

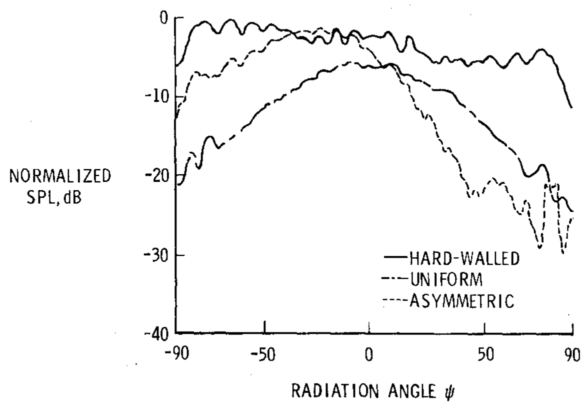

Fig. 7 Directivity patterns for a (4,0) mode, $ka=6.0$, $\ell/a=1$.

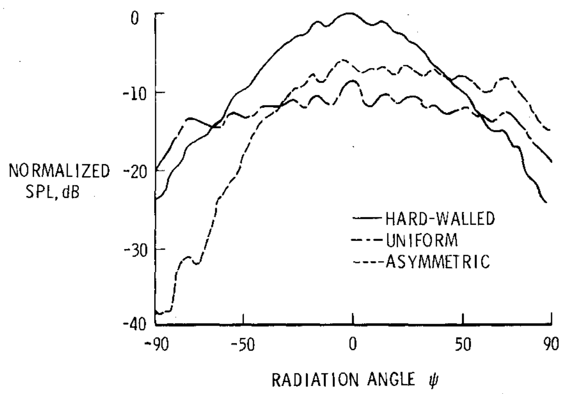

Fig. 8 Directivity patterns for a (0,0) mode, $ka=3.5$, $\ell/a=1$.

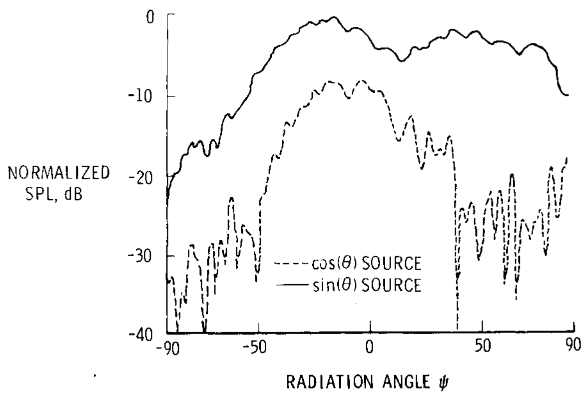

Fig. 9 Effects of a rotation of a (1,0) source by 90 deg on the far-field radiated pressure, $ka=4.9$, $\ell/a=1$.

When the incident wave is just above cut-on, parameters other that the distribution of impedance become equally significant. Figure 7 shows the measured radiation patterns when the incident wave is a (4,0) mode at $ka=6.0$. For this example the cut-off ratio is 1.17 and the length of liner is $\ell/a=1$. The first characteristic observable from Fig. 7 is the lack of a null in the radiation patterns of the uniform cases at $\psi=0$. This result is due to the fact that although the signal isolation between the (4,0) and the (0,1) and (0,0) modes incident in the hard-walled duct is greater than 15 dB, the (0,1) and the (0,0) modes dominate the radiation patterns at small angles of ψ. This characteristic is due to a combination of the (0,1) and (0,0) modes having a much lower reflection coefficient, lower decay rate in the lined case, and enhanced directivity at small angles of ψ than the (4,0) mode. This is also apparent for the uniformly lined case of Fig. 7, where the radiation pattern is very similar to that of a (0,0) mode at the same frequency. The effect of the liner in this case has been to suppress the main radiation lobes of the (4,0) and (0,1) modes. The results of Fig. 7 show that for this case the asymmetric liner provides improved performance over the uniform liner in the positive ψ sector of the radiated field. At negative angles of ψ, the radiated levels are much greater than the uniform case.

The results of Fig. 8 are for an incident wave corresponding to the (0,0) or plane mode at $ka=3.5$. The length of lined section is $\ell/a=1$. When the impedance distribution is asymmetric, a strong distortion is apparent in the radiated field. This distortion can be attributed to the asymmetric nature of the pressure field in the lined section as discussed in Ref. 8. Furthermore the distortion is such that the levels radiated are less in the far-field sector on the same side as the lined region of the duct, contrary to what is physically expected from ray theory. This behavior, while not typical, underlines the complexity of the radiation process from asymmetrically lined ducts. Similar characteristics have been observed in theoretical calculations and were found to be due to the presence of two or more modes of approximately equal strength (energy) and decay rates in the lined section of the duct system and subsequent interference effects in the far-field. The results of Fig. 8 show that for this case the asymmetric liner provides greater attenuation than the uniformly lined case in a significant sector of the radiated field.

By accident or design an asymmetrically lined duct has an impedance distribution that varies with circumferential angle θ. Thus, it is logical that the shape of the radiation pattern of

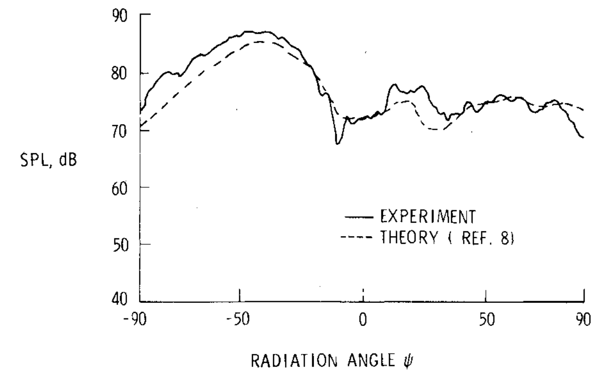

Fig. 10 Comparison of experimental and theoretical directivity patterns for a (3,0) mode, $ka=7.2$, $\ell/a=2$.

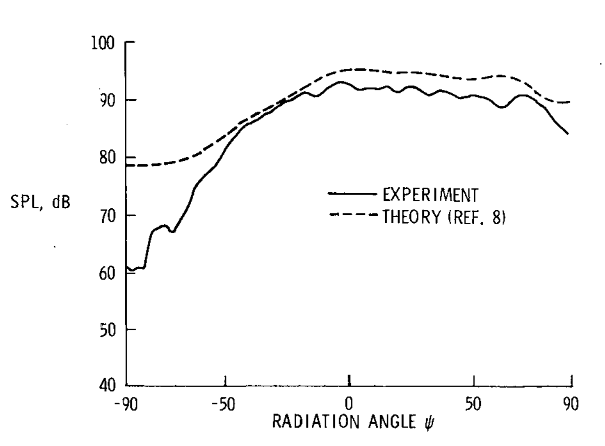

Fig. 11 Comparison of experimental and theoretical directivity patterns for a (0,0) mode, $ka=3.5$, $\ell/a=1$.

Table 1 Comparison of reflection coefficients and phase changes

Mode	ℓ/a	ka	Cut-off ratio	$\|R\|$ Exp	$\|R\|$ Theory	$\Delta\phi$ (deg) Exp	$\Delta\phi$ (deg) Theory
(0,0)	1	3.5	—	0.11	0.16	−95.0	95.7
(1,0)	2	4.9	2.67	0.20	0.05	−132.1	26.5
(2,0)	2	3.6	1.18	0.46	0.43	−157.6	−165.1
(3,0)	2	7.2	1.72	0.11	0.04	−75.7	−76.7
(4,0)	1	6.0	1.13	0.50	0.52	162.4	44.2

the asymmetrically lined duct will depend strongly on the angular location of the standing wave source relative to the liner distribution. The directivity patterns for a $\cos(\theta)$ and a $\sin(\theta)$ source at $ka=4.9$ radiating from the asymmetrically lined duct of length $\ell/a=1$ are plotted in Fig. 9. The two cases correspond to a rotation of a (1,0) source by 90 deg. The results show that a rotation of the $\cos(\theta)$ source causes a marked drop in the radiated levels at all angles of ψ. As discussed in Ref. 8, this effect is due to the coupling of the modes in the asymmetrically lined section being directly dependent upon the angular location of the source. When the $\cos(\theta)$ source is rotated by 90 deg, the least attenuated mode in the lined section is theoretically decoupled from the incident wave and the radiated levels correspondingly fall. Thus, in a physical situation with a slowly rotating standing wave source (this may result from a varying interference between positive and negative spinning modes) radiating through an asymmetrically lined duct, the radiated rms levels will vary periodically at a frequency related to the circumferential modal number of the source and the periodicity of the liner distribution.

Figures 10 and 11 show a comparison between the experimental results presented previously in Figs. 6 and 8 and theoretical predictions calculated using the analysis described in Ref. 8. Values of impedance of the liner material measured previously (see Fig. 4) were used in the theory. As can be seen from Figs. 10 and 11, the agreement between theory and experiment is good, especially as the levels presented are absolute values at the measuring point in the radiated field for a prescribed amplitude of the source. The errors apparent at large angles of ψ are most likely due to be the finite size of the experimental baffle and the proximity of the wall of the anechoic chamber. Reasons for other differences between the two results were twofold. First, the difference may be due to contributions of other modes present in the incident wave field, but not accounted for by theory. Second, the impedance of the liner material was measured at normal incidence and is assumed to be locally reacting. In the practical situation, however, the liner material is likely to display some extended reaction effects and the acoustic field has an axial component at the liner face. On the whole, this degree of agreement was found for all the cases investigated.

Table 1 presents a comparison between the experimentally measured and theoretically predicted amplitude reflection coefficients and phases for the asymmetric liner with sources of increasing modal number at various selected frequencies. Generally the reflection coefficient of the asymmetrically lined duct was measured to be high near the cut-on frequency of the source wave and falls rapidly with increasing frequency. This behavior is well illustrated in the theoretical results of Ref. 8. The agreement between the experimental and theoretical values of reflection coefficient can be seen to be reasonable especially at higher values of reflection coefficient when the signal-to-noise ratio of the reflected wave is improved. However, the agreement for the magnitude of phase change is poor. The reason for this discrepancy was thought to be due to inaccuracies in the measure phase speed in the duct due to axial temperature gradients. The measured phase referenced to the inlet plane was found to be extremely sensitive to small changes in temperature when the phase speed of the mode was high.

Concluding Remarks

An experiment was conducted in the NASA Langley Anechoic Noise Facility to study the radiation of sound from an asymmetrically lined duct with various incident hard-walled modes. A comparison between the experimental results and those obtained using a previously developed theory was made. In general, good agreement was found for the radiation directivity patterns and amplitude reflection coefficients of the asymmetrically lined outlet, while poor agreement was found for the reflected wave phase change.

The experimental results confirm that an asymmetry in the circumferential distribution of liner impedance can cause a significant distortion in the acoustic field radiated from the outlet when compared to the behavior of baseline hard-walled and uniformly lined ducts. Although in most cases observed the distortion is such that the radiated levels are less in that part of the radiation field diagonally opposite the lined section, in isolated cases this behavior is reversed. It was also shown that the source orientation was important to the radiation characteristics. On the whole, for the cases in which significant distortion in the radiated field was observed, the asymmetric liner performed comparably with the uniform liner in reducing noise levels in half the radiated field.

The results presented indicate some possibility for the use of asymmetric liners for reducing the levels of sound radiated towards the ground from the inlets of turbofan engines. They also underline the importance of including radiation effects into the design of turbofan inlet liners.

Acknowledgments

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